PHENOMENOLOGICAL GUIDE TO PHYSICS BEYOND THE STANDARD MODEL [1]

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Abstract

Various aspects of physics beyond the Standard Model are discussed from the perspective of the fantastic phenomenological success of the Standard Model, its simplicity and predictive power.

Content

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1 Introduction

The Standard Model is a successful theory of interactions of quarks and leptons at energies up to about hundred GeV. Despite that success it is widely expected that there is physics beyond the Standard Model, with new characteristic mass scale(s), perhaps up to, ultimately, a string scale.

The expectation is motivated by several fundamental questions that remain unanswered by the Standard Model. The most pressing one is better understanding of the mechanism of the electroweak symmetry breaking. The origin of flavour and of the pattern of fermion masses and of CP violation also remain beyond its scope. Moreover, we know now that the physics of the Standard Model cannot explain the baryon asymmetry in the Universe. And on the top of all that come two recent strong experimental hints for physics beyond the Standard Model, that is very small neutrino masses and the presence of dark matter in the Universe. The list can be continued by including dark energy and inflation.

The Standard Model does not explain the scale of the electroweak symmetry breaking. It is a free parameter of the theory, taken from experiment. Moreover, once we accept the point of view that the Standard Model is only an effective "low energy" theory which is somehow cut-off at a mass scale M, and if $M \gg M_{W,Z}$, the electroweak symmetry breaking mechanism based on use of an elementary Higgs field is unstable against quantum corrections (this is the so-called hierarchy problem).

Many different extensions of the Standard Model have been proposed to avoid the hierarchy problem and, more ambitiously, to calculate the scale of the electroweak breaking in terms of, hopefully, more fundamental parameters. Some extensions give the complete Standard Model, with one Higgs doublet, as their low energy approximation in the sense of the Appelquist - Carazzone decoupling and in some others the mechanism of the electroweak symmetry breaking cannot be decoupled from the bigger theory. The general idea is that the bigger theory has some characteristic mass scale M only order of magnitude bigger than $M_{W,Z}$, which plays the role of a cut-off to the electroweak sector. All those extensions of the Standard Model have distinct experimental signatures. The experiments at the LHC will, hopefully, shed more light on the mechanism of the electroweak symmetry breaking and will support one of those (or still another one ?) directions.

One approach is based on low energy supersymmetry. The scale M is identified with the mass scale of supersymmetric partners of the Standard Model particles. Supersymmetry is distinct in several very important points from all other proposed solutions to the hierarchy problem. First of all, it

provides a general theoretical framework which allows to address many physical questions. Supersymmetric models, like the Minimal Supersymmetric Standard Model or its simple extensions satisfy a very important criterion of "perturbative calculability". In particular, they are easily consistent with the precision electroweak data. The Standard Model is their low energy approximation in the sense of the Appelquist - Carazzone decoupling, so most of the successful structure of the Standard Model is built into supersymmetric models. Unfortunately, there are also some troublesome exceptions: there are new potential sources of Flavour Changing Neutral Current (FCNC) transitions and of CP violation, and baryon and lepton numbers are not automatically conserved by the renormalizable couplings. But even those problems can at least be discussed in a concrete way. The quadratically divergent quantum corrections to the Higgs mass parameter (the origin of the hierarchy problem in the Standard Model) are absent in any order of perturbation theory. Therefore, the cut-off to a supersymmetric theory can be as high as the Planck scale and "small" scale of the electroweak breaking is still natural. But the hierarchy problem of the electroweak scale is solved at the price of a new hierarchy problem of the soft supersymmetry braking scale versus the Planck (string) scale. Spontaneous supersymmetry breaking and its transmission to the visible sector is a difficult problem and a fully satisfactory mechanism has not yet been found. Again on the positive side, supersymmetry is not only consistent with Grand Unification of elementary forces but, in fact, makes it very successful. And, finally, supersymmetry is needed for string theory.

All other extensions of the electroweak theory proposed as solutions to the hierarchy problem rely on an onset of some kind of strong dynamics at energy scales not much higher than the electroweak scale. In some of them, like Higgless models with dynamical electroweak symmetry breaking or strong gravity in large extra dimensions, the strong dynamics is simply a cut-off directly to the electroweak sector and appears already at $\mathcal{O}(1\,\mathrm{TeV})$. In models with the Higgs boson as a pseudo-Goldstone boson (e.g. Little Higgs models) and models with gauge fields present in extra dimensions the cut-off scale M is identified with the characteristic scale of new perturbative physics, e.g. with the scale of breaking of some global symmetry or with the radii of extra dimensions. However, since those models are non-renormalizable and, moreover, in the bigger theory the quadratic divergences to the scalar mass parameter are absent typically only at one loop level, new physics itself has to be cut-off by some unknown strong dynamics at a scale one or two orders of magnitude higher than the M. Generally speaking, there is no Appelquist - Carazzone decoupling of new physics and the precision tests of

such a version of the electroweak theory are not possible at the same level of accuracy as in the renormalizable Standard Model.

It is clear that models with early onset of strong dynamics cannot be easily, if at all, reconciled with Grand Unification. Also, they are very strongly constrained by precision electroweak data. There have been constructed models that work but simple models are usually ruled out. Moreover, various aspects of flavour physics are often very obscure.

Spontaneous symmetry breaking in the condensed matter physics and in QCD is due to some collective effects. In supersymmetric models, such effects are presumably responsible for spontaneous breaking of supersymmetry and, in consequence, for the generation of soft mass terms. However, the electroweak symmetry breaking is driven by perturbative quantum corrections, generated by the large top quark Yukawa coupling, to the scalar potential. In the Little Higgs models, the Higgs boson is a Goldstone boson of a bigger spontaneously broken global symmetry group. The Higgs potential needed for the electroweak symmetry breaking is also given by quantum corrections, with important contribution from the top quark Yukawa coupling. Thus, one thing many models have in common is that the electroweak symmetry is broken by perturbative quantum effects and linked to the large mass of the top quark.

At present, all extensions of the Standard Model remain speculative and none is fully satisfactory. Remembering the simplicity, economy and success of the Standard Model, one may wonder if in our search for its extensions shouldn't the Hipocrates principle *Primum non nocere* play more important role than it does. Indeed, various new ideas offer surprisingly low ratios of benefits to losses. It is, therefore, appropriate to begin by reviewing the basic structure of the Standard Model that underlies its success. It is likely that it gives us important hints for the physics beyond.

2 The Standard Model

2.1 Basic structure

The underlying principles of the electroweak theory are:

1. local $SU(2)_L \times U(1)_Y$ gauge symmetry and electroweak unification

2. spontaneous breaking of $SU(2)_L \times U(1)_Y$ gauge symmetry to $U(1)_{EM}$, by the Higgs mechanism with one Higgs doublet

3. matter content (chiral fermions)

4. renormalizability

Massless chiral fermions are the fundamental objects of matter: left-handed, with helicity $\lambda = -1/2$, and right-handed, with helicity $\lambda = 1/2$. It is so because parity and charge conjugation are not the symmetries of our world. The left-handed fermions carry different weak charges from the right-handed fermions. Chiral fermion fields are two-component (Weyl) spinors (see e.g. [1]):

$$SU(2)_L \text{ doublets}$$

$$q_1 \equiv \begin{pmatrix} u \\ d \end{pmatrix} \qquad q_2 \equiv \begin{pmatrix} c \\ s \end{pmatrix} \qquad q_3 \equiv \begin{pmatrix} t \\ b \end{pmatrix} \tag{1}$$

$$l_1 \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad l_2 \equiv \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad l_3 \equiv \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{2}$$

with the electric charge and the hypercharge ($Y = Q - T_3$) assigned as below

	u	d	ν	e
Q	2/3	-1/3	0	-1
Y	1/6	1/6	-1/2	-1/2

These are left-handed chiral fields in the representation $(0, 1/2)$ of the $SL(2, C)$, each describing two massless degrees of freedom: a particle with the helicity $\lambda = -1/2$ and its antiparticle with $\lambda = +1/2$. (The chiral fields can also be written as four-component spinors (see e.g. [1]) but in the following we shall be using the Weyl notation).

Right-handed fields [$(1/2, 0)$ of $SL(2, C)$] in the same representations of $SU(2)_L \times U(1)_Y$ as the left-handed fields (1) and (2) do not exist in Nature. Instead, we have

$$SU(2)_L \text{ singlets}$$

$$\begin{array}{ccc} u_R, & c_R, & t_R \\ d_R, & s_R, & b_R \\ e_R, & \mu_R, & \tau_R \end{array}$$

in $(1, +2/3)$, $(1, -1/3)$ and $(1, -1)$ of $SU(2)_L \times U(1)_Y$, respectively. These are right-handed chiral fields in the $(1/2, 0)$ representation of the group

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$SL(2,C)$. For constructing a Lorentz invariant Lagrangian, it is more convenient to take as fundamental fields only the left-handed chiral fields. Thus, we introduce left-handed chiral fields, e.g

$$u^c, \quad c^c, \quad t^c \tag{3}$$

in $(1, -2/3)$ of $SU(2)_L \times U(1)_Y$, such that

$$\bar{u}^c \equiv CPu^c(CP)^{-1} = u_R \tag{4}$$

Indeed, CP transformation results in the simultaneous change of chirality and charges (representation $R \to R^*$ for internal symmetries). Moreover, we see that the electric charge $Q = T_3 + Y$ satisfies, e.g.

$$Q_{u^c} = -2/3 = -Q_u \tag{5}$$

and the two left-handed fields u and u^c become charge conjugate to each other when $U_{EM}(1)$ remains the only unbroken symmetry:

$$Cu^cC^{-1} = u \tag{6}$$

We note that the matter chiral fields of the SM do not include a right-handed neutrino field ν_R in $(1, 1)$ of $SU(2)_L \times U(1)_Y$ (such a charge assignment preserves the relation $Q = T_3 + Y$) or equivalently, a left-handed field ν^c such that

$$\nu_R = CP\nu^c(CP)^{-1} \tag{7}$$

but we can supplement the Standard Model with such a particle, if useful.

The breaking of the electroweak symmetry is generated by the potential of the Higgs doublet $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ with the hypercharge $Y = +1/2$:

$$V = m^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2 \tag{8}$$

When $m^2 < 0$ is chosen, the Higgs doublet acquires the vacuum expectation value. Indeed, the minimum of the potential is for

$$\langle H^\dagger H \rangle = -\frac{m^2}{\lambda} \equiv \frac{v^2}{2} \tag{9}$$

By $SU_L(2)$ rotation we can always redefine the vacuum so that only the VEV of the lower component of the Higgs doublet is non-zero. The $SU_L(2) \times U_Y(1)$ symmetry is then broken down to $U'(1)$ which is identified with $U_{EM}(1)$

with $Q = T^3 + Y$ because $(T^3 + Y)\begin{pmatrix} 0 \\ v \end{pmatrix} = 0$. The parameters m and λ are free parameters of the Standard Model. Equivalently, the scale of the electroweak symmetry breaking is not predicted by the theory and must be taken from experiment.

2.2 Fermion masses

Higgs doublets (and only doublets) have $SU(2)_L \times U(1)_Y$ invariant renormalizable couplings to the chiral fermions of the Standard Model. For the charged fermions, we can write down the following Yukawa couplings:

$$\mathcal{L}_{Yukawa} = -Y_l^{BA} H_i^* l_{iA} e_B^c - Y_d^{BA} H_i^* q_{iA} d_B^c - Y_u^{BA} \epsilon_{ij} H_i q_{iA} u_B^c + hc \quad (10)$$

where i is the $SU(2)_L$ index and A, B are generation indices. We use the fact that the two-dim representation of $SU(2)$ is real and $i\tau_2 H$ transforms as H^*, i.e. as $2^*(\equiv 2)$ of $SU(2)$. Therefore, $(i\tau_2 H q) = \epsilon_{ij} H_i q_j$ is also an invariant of $SU(2)$. After spontaneous breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$ by the Higgs boson vacuum expectation value v we obtain the Dirac masses

$$\mathcal{L}_{mass} = -v(Y_l^{BA} e_A e_B^c + Y_d^{BA} d_A d_B^c + Y_u^{BA} u_A u_B^c) + hc \quad (11)$$

However, at the level of the full, $SU(2)_L \times U(1)_Y$ invariant theory, there is no renormalizable term that would give neutrino mass. It is so because ν^c is absent from the spectrum of the SM. Thus, in the SM, neutrinos are massless.

The interactions (10,11) are written in some "electroweak" basis defined by eigenvectors of the $SU(2)_L \times U(1)_Y$ symmetry group. In such a basis, both the fermion masses and the Yukawa couplings are in general non-diagonal in the flavour indices (A, B). However, we can introduce another set of fields (say, primed fields) describing physical particles (mass eigenstates). The flavour of the primed fields is defined in the mass eigenstate basis. The two sets of fields are related to each other by unitary transformations:

$$u = U_L u' \qquad d = D_L d'$$

$$u^c = u'^c U_R^\dagger \quad d^c = d'^C D_R^\dagger$$

$$e = E_L e'$$

$$e^c = e'^c E_R^\dagger$$

$$(12)$$

which, of course, do not commute with the $SU(2)_L \times U(1)_Y$ gauge transformations and can be performed only after the spontaneous breakdown of the gauge symmetry. In eq.(12), the fields u, d, e denote three-dimensional vectors in the flavour space.

The transformations (12) diagonalize the mass terms and the Yukawa couplings defined by (10). After diagonalization we can combine the chiral fields into Dirac fields. The weak currents can be expressed in terms of the physical (mass eigenstates) fields:

$$
\begin{aligned}
J_\mu^- &= \sum_{A,B} \bar{u}'_A \bar{\sigma}_\mu (V_{CKM})_{AB} d'_B \\
&+ \sum_A \bar{\nu}'_A \bar{\sigma}_\mu e'_A
\end{aligned}
\tag{13}
$$

where the Cabibbo-Kobayashi-Maskawa matrix $V_{CKM} = U_L^\dagger D_L$. Note that the lepton current is diagonal in flavour (defined in the charged lepton mass eigenstate basis) because the <u>massless</u> neutrino field can be redefined by the transformation $\nu_A = E_{AB}^L \nu'_B$ where E_{AB}^L is the transformation diagonalizing the charged lepton mass matrix (see (12)). Thus, for the lepton current, $V_{CKM} = E^{L\dagger} E^L = 1$.

It is important to remember that in the SM (with only one Higgs doublet) the Yukawa couplings to the physical Higgs boson (and, in fact, also the couplings to the Z^0) and the mass terms are diagonalized by the same unitary rotations. So they are flavour diagonal. The only source of flavour non-conservation resides in V_{CKM}. In particular, not only the global lepton number but also each flavour lepton number is separately conserved.

2.3 Approximate custodial symmetry of the Standard Model and the precision electroweak data

The Higgs sector of the SM is invariant under global $SO(4)$ symmetry acting on four real components of the complex doublet. The group $SO(4) \simeq SU(2)_L \times SU(2)_R$ and the Higgs doublet can be written as a 2×2 matrix Φ

$$
\Phi = \begin{pmatrix} H^+ & H^{0*} \\ H^0 & -H^- \end{pmatrix} ,
\tag{14}
$$

which transforms as $(\mathbf{2}, \mathbf{2})$ of the latter group (whose first factor is just the gauged weak isospin group):

$$
\Phi \longrightarrow \Phi' = U_L \Phi U_R .
\tag{15}
$$

The vacuum expectation value of the Higgs field breaks $SU(2)_L \times SU(2)_R$ to its diagonal subgroup called "custodial" $SU(2)$ acting on the three would-be Goldstone bosons G^a:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} e^{\frac{iG^a \tau^a}{v}} \begin{pmatrix} 0 \\ v + h^0 \end{pmatrix} \tag{16}$$

In the rest of the electroweak Lagrangian the $SU(2)_R$ subgroup and therefore also the custodial $SU(2)$ symmetry is broken by the Yukawa interactions and by the $U(1)_Y$ coupling. However, the custodial symmetry is still seen at the tree level since by the Higgs mechanism it ensures that the gauge bosons W^+, W^- and W^0 of $SU(2)_L$ transform as a custodial triplet. In consequence, the ratio of the strength of charged and neutral current interaction at the tree level is equal to one

$$\rho \equiv \frac{M_W^2}{\cos^2 \theta_W M_Z^2} = 1 \ , \tag{17}$$

where $\cos^2 \theta_W = g_2^2/(g_2^2 + g_1^2)$.

This relation is consistent to a very good approximation with experimental data but this is not the end of the success of the SM. Since the rest of the Lagrangian violates the custodial symmetry, there are quantum corrections to the relation $\rho = 1$. Since the global $SU(2)_L \times SU(2)_R$ symmetry of the Higgs sector fixes the structure of counterterms in the scalar potential, quantum corrections to the relation $\rho = 1$ must be finite! In one-loop approximation one gets

$$\Delta\rho \equiv \rho - 1 = \frac{3g_2^2}{64\pi^2} \frac{m_t^2 - m_b^2}{M_W^2} - \frac{g_1^2}{64\pi^2} \frac{11}{3} \ln \frac{M_h^2}{M_W^2} \ . \tag{18}$$

The first term is of the order of 1% and, is in perfect agrrement with the precision electroweak data. Thus, fits to the data give $M_h \sim \mathcal{O}(M_W)$ although logarithmic dependence of $\Delta\rho$ on M_h does not allow for precise determination of this mass. We shall discuss later on the importance of the Higgs particle mass for various extensions of the SM.

We conclude that the approximate custodial symmetry of the SM is in fantastic agreement with experimental data. Any alternative mechanism of the electroweak symmetry breaking or any extension of the SM must not violate the custodial symmetry of the electroweak interactions. Furthermore, we see that the renormalizable SM with one Higgs doublet, has very strong predictive power which allows for its precision tests at the level of one per mille.

2.4 GIM mechanism and the suppression of FCNC and CP violating transitions

It is well established experimentally that the amplitudes for processes such as e.g. $K^0(d\bar{s})$-$\bar{K}^0(\bar{d}s)$ mixing caused by electroweak interactions, $\langle K^0|\mathcal{H}_{\text{weak}}|\bar{K}^0\rangle$, are strongly suppressed in comparison with the amplitudes for the charged current transitions like $n \to pe^-\bar{\nu}_e$ or $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. A good measure of the mixing is the mass difference between neutral kaon mass eigenstates $|K_L^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle)$ and $|K_S^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$ (we neglect here even smaller CP violation): $\Delta M_K = 3 \times 10^{-12}$ MeV and is suppressed by factor 10^6 compared to what one could expect for a generic electroweak transition. This fact finds a very elegant explanation in the Standard Model. The charged current transitions shown in Fig. 1 are present at the tree level whereas it follows from the structure of the theory that the diagrams shown in Fig. 2 are absent. In the diagrams the quark fields are of course mass eigenstate fields and the couplings are obtained by rotating from an electroweak basis, in which the theory is formulated, to the mass eigenstate basis, in which the quark flavour is defined.
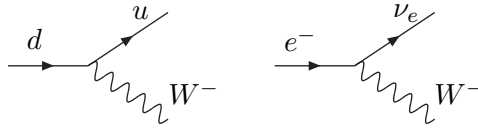

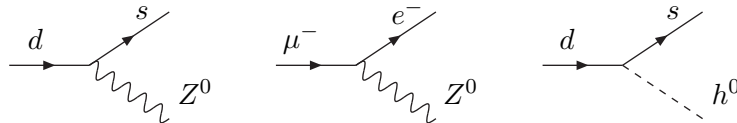Figure 1: Charged current transitions in the Standard Model.

Figure 2: Neutral current transitions absent at the tree level in the SM.

It is obvious that the Z^0 couplings are flavour diagonal. More interesting is the absence in the SM of the scalar flavour changing neutral currents. This result follows from the fact that in the SM there is only one Higgs doublet. Because of that, diagonalizing the fermion mass matrices one simultaneously

obtains also diagonal Yukawa couplings to the physical Higgs boson. In models with more Higgs doublets, additional discrete symmetry has to be imposed to ensure that only one Higgs doublet couples to the quarks of the same charge. In the minimal supersymmetric model two doublets are needed for supersymmetric theory but (by the holomorphicity of the superpotential) only one Higgs doublet can couple to the same charge quarks.
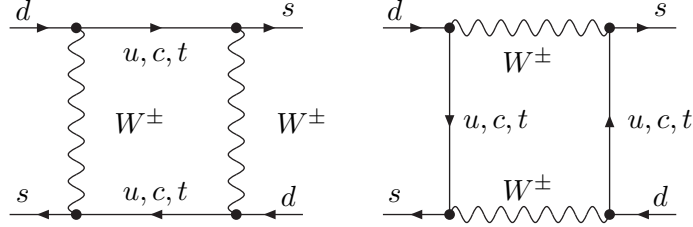

Figure 3: Leading SM contribution to K^0-$\bar{K}^0$ mixing.

The absence of flavour non-diagonal neutral currents at the tree level is not sufficient to account for the observed suppression of processes like kaon mixing or $b \to s\gamma$. For example the 1-loop diagrams shown in Fig. 3 generate the $K^0(d\bar{s})$-$\bar{K}^0(\bar{d}s)$ transitions and the coefficient C in the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = C \ (\bar{s}_L\gamma_\mu d_L)(\bar{s}_L\gamma_\mu d_L) \tag{19}$$

describing their contribution (in the limit of external quark momenta small compared to M_W) is generically of order $C \sim \frac{\alpha^2}{M_W^2} \sim \alpha \ G_F$. However, in the SM the sum of all such contributions is suppressed by a factor $\sim 10^{-4}$ due to the so-called (generalized to 3 generations of quarks) Glashow-Illiopoulos-Maiani mechanism. The coefficient C generated by diagrams of Fig. 3 (and the diagrams in which one or both $W^\pm$ are replaced by the unphysical would-be Goldstone bosons $G^\pm$) is finite and has dimension mass^{-2}. The whole effective Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \frac{1}{2}\left(\frac{g_2}{\sqrt{2}}\right)^4 \sum_{i,j=u,c,t} V_{is}^\star V_{id} V_{js}^\star V_{jd} \tag{20} \\ &\times \int \frac{d^4q}{(2\pi)^4} \frac{[\overline{\Psi}_s\gamma_\mu P_L(\slashed{q}+m_{q_i})\gamma_\nu P_L\Psi_d][\overline{\Psi}_s\gamma^\nu P_L(\slashed{q}+m_{q_j})\gamma^\mu P_L\Psi_d]}{(q^2-m_{q_i}^2)(q^2-m_{q_j}^2)(q^2-M_W^2)^2} \end{aligned}$$

The top quark contribution is suppressed by the smallness of the product $V_{ts}^*V_{td}$. The rest contributes to the coefficient C in (19)

$$
\begin{aligned}
A_{uc} \quad \sim \quad & \left(\frac{g_2}{\sqrt{2}}\right)^4 \frac{1}{M_W^2} \sum_{i,j=u,c} V_{is}^\star V_{id} V_{js}^\star V_{jd} \left[1 + \mathcal{O}\left(\frac{m_{q_i}^2}{M_W^2}, \frac{m_{q_j}^2}{M_W^2}\right)\right] \\
\sim \quad & \alpha\, G_F \left\{ (V_{ts}^\star V_{td})^2 + \mathcal{O}\left(\sum_{i,j=u,c} V_{is}^\star V_{id} V_{js}^\star V_{jd} \frac{m_q^2}{M_W^2}\right) \right\}
\end{aligned}
\tag{21}
$$

where in the last line, unitarity of the CKM matrix has been used: $V_{us}^\star V_{ud} + V_{cs}^\star V_{cd} = -V_{ts}^\star V_{td}$.

From this example it is clear that for the empirical pattern of quark masses and mixing angles there is strong suppression of FCNC in the SM. It is much stronger than "naturally" expected.

The predictions of the SM for the FCNC transitions are in very good agreement with experimental data. This is also true for CP violation. The only source of CP violation in the SM is the phase of the CKM matrix. As a result, the effects of CP violation in the kaon system, in which they were first observed, are proportional to the masses of the light quarks and small CKM mixing angles and hence very small (this is not so for the B-meson systems in which CP violation is probed by present experiments).

The strong suppression of the FCNC and CP violating transitions, so nicely consistent with the SM is a big challenge for any of its extension. This is easy to understand on a qualitative basis. Let us suppose that new physics contributes to such transitions at 1-loop level (any contribution at the tree level would be a total disaster!) with the couplings of order of the strong coupling constant $\alpha_s \approx 0.12$ and with the scale M of the particle masses in the loop. Then

$$
\Delta C \sim \frac{\alpha_s^2}{M^2} = \frac{\alpha^2}{M_W^2} \left(\frac{\alpha_s}{\alpha}\right)^2 \left(\frac{M_W}{M}\right)^2
\tag{22}
$$

Thus, for such contributions to be comparable or smaller than the SM one, the new physics scale M has to be higher than $10^3 M_W \sim 100$ TeV. If the new couplings are of the order of α then we get $M \gtrsim 10$ TeV. If the scale of new physics extension of the SM is below these limits, the new physics must somehow control the flavour effects!

2.5 Baryon and lepton number conservation

The principles (1) - (4) <u>imply</u> global $U(1)$ symmetries of the theory: baryon and lepton number conservation $\Delta B = \Delta L = 0$. In fact, for leptons

the implication is even stronger, namely $U_e(1) \times U_\mu(1) \times U_\tau(1)$ is a global symmetry of the electroweak Lagrangian and the lepton flavour numbers are separately conserved: $\Delta L_e = \Delta L_\mu = \Delta L_\tau = 0$. For quarks, quark mixing explicitly breaks quark flavour $U(1)$'s and only the total baryon number is conserved.

The conservation of the baryon and lepton numbers by the renormalizable couplings of the Standard Model is beautifully consistent with experimental limits on the life time of the proton , $\tau_p \gtrsim 10^{33}$ years, and on the branching ratios for the lepton flavour violating decays, e.g. $BR(\mu \to e\gamma) \lesssim 10^{-11}$. Proton decay and lepton flavour violating decays occur, if at all, many orders of magnitude less frequently than generic electroweak processes. Actually, in the Standard Model those conservation laws are violated by chiral anomaly. The diagrams shown in Fig. 4 where $j^a_{\mu L}$'s are $SU(2)_L$ gauge currents and $j^{L,B}_\mu$ is the baryon or lepton current of the $U(1)$ global symmetries gives (insisting on the conservation of the gauge currents [1])

$$\partial^\mu j^B_\mu \propto (\mathrm{Tr}B) \sum_{a=1}^{3} W^a_{\mu\nu} \tilde{W}^{a\mu\nu}$$

$$\partial^\mu j^L_\mu \propto (\mathrm{Tr}L) \sum_{a=1}^{3} W^a_{\mu\nu} \tilde{W}^{a\mu\nu} \qquad (23)$$
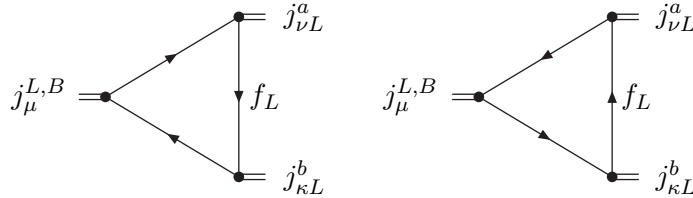

Figure 4: Anomalies in the B and L currents in the SM.

Only $SU_L(2)$ doublets contribute to the traces, so they do not vanish, and $W^a_{\mu\nu}$ is the $SU(2)_L$ field strength. Non-perturbative effects give, in general a non-zero condensate $W^a_{\mu\nu}\tilde{W}^{\mu\nu}$ (topological baryon and lepton number non-conservation) but the effect is totally negligible at zero temperature. At non-zero temperature, the topological baryon and lepton number non-conservation is enhanced and can play important physical role because the $(B-L)$ current is anomaly free: $\partial^\mu j^{B-L}_\mu \propto \mathrm{Tr}(B-L) = 0$ and the quantum

number $B - L$ is conserved. Thus in the presence of some hypothetical perturbative lepton number and CP violation, topological effects may convert leptogenesis into baryogenesis.

Incidentally, with the right-handed neutrino included in the spectrum, the diagrams in Fig. 5 do not give any anomaly, neither, and the $(B - L)$ symmetry can, therefore, be gauged.
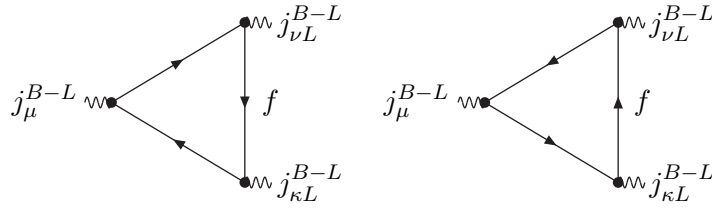

Figure 5: $[U(1)_{B-L}]^3$ anomaly.

3 Hints from the Standard Model for its extensions

3.1 Is the effective low energy electroweak theory indeed the renormalizable Standard Model?

In the construction of the SM Lagrangian we have been, so far, guided by its renormalizability. Accepting the fact that the SM is only an effective theory one may wonder, however, how important is its renormalizability. Unitarity and symmetries are certainly more fundamental requirements and indeed e.g. the physics of pions is described by a non-renormalizable effective low energy theory (non-linear σ-model). It is, therefore, useful to recall the main differences between the two classes of quantum field theories.

In a renormalizable theory its cut-off can be taken to infinity and the whole UV sensitivity is hidden in a finite number of free parameters, the same at any order of perturbation expansion. Calculations with arbitrary precision are, therefore, possible with a fixed number of parameters whose values can be determined from the experimental data. If some theory gives as its low energy approximation a renormalizable theory, then according to the Appelquist-Carazzone decoupling theorem, the effects of heavy degrees of freedom characterized by a mass scale M show up only as corrections in the form of higher dimensional operators allowed by the symmetries of the

renormalizable theory:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{renormalizable}} + \mathcal{O}\left(\frac{1}{M^n}\right) \, O_{4+n} \tag{24}$$

This is a window to new physics (if such corrections are needed by experiment) even if we do not know the theory at the scale M.

It is worth putting the view at the Standard Model as an effective low energy theory into the better known perspective. We know now that Quantum Electrodynamics (QED) is a renormalizable theory and at the same time it is the low energy approximation to the electroweak theory. Its renormalizability means calculability with arbitrary precision. But it is only an effective theory so we know that its predictions disagree with experiment at the level $\sim \mathcal{O}(E/M_{\text{W}})$, where the energy E is the characteristic energy for a given process. For example, let us have a look at the lepton magnetic moment. It gets contributions from the diagrams depicted in Fig. 6. Thus, for the
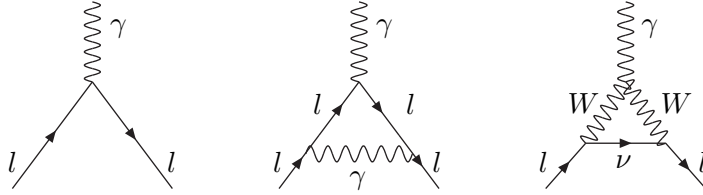

Figure 6: One loop contributions to the anomalous lepton magnetic moment in the SM.

nonrelativistic effective interaction with the magnetic field we get

$$\mathcal{H}_{\text{eff}} = \frac{e}{2m_l}\boldsymbol{\sigma}\cdot\mathbf{B}(1 + \frac{\alpha}{2\pi} + \mathcal{O}\left(\alpha\frac{m_l^2}{M_W^2}\right) + \ldots) \tag{25}$$

where the role of the energy scale is played by the lepton mass m_l. The "weak" correction is calculable in the full electroweak theory, but at the level of QED as an effective theory it has to be added as a new non-renormalizable (but $U(1)_{EM}$ invariant) interaction

$$\mathcal{L}_{\text{eff}} = \frac{m_l}{M^2}\bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} \qquad \text{(dim 5)} \tag{26}$$

This would have been a way to discover weak interactions (and to measure the weak scale) in purely electromagnetic processes: we extend QED
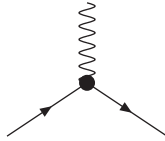
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Figure 7: Effective photon-lepton vertex.

to a non-renormalizable theory by adding higher dimension operators and look for their experimental manifestation in purely electromagnetic processes once the experimental precision is high enough. Luckily enough for us, effective QED may also contain other than (26) non-renormalizable corrections, $U(1)_{EM}$ invariant but violating the conservation of quantum numbers that are accidentally conserved in QED, for instance flavour. Such corrections manifest themselves as different type of interactions - weak interactions - and were easy to discover experimentally. Similarly, among many possible non-renormalizable corrections to the SM which respect the $SU(2)_L \times U(1)_Y$ gauge symmetry there are such that violate e.g. the lepton and/or baryon number conservation or give Majorana masses to neutrinos. We shall discuss them in the following.

In a non-renormalizable theory one either has to keep explicit logarithmic cut-off dependence ($\frac{1}{16\pi^2} \ln \frac{\Lambda}{\mu}$ where μ is some low energy scale) or the number of counterterms (i.e. the number of free parameters of the theory) must increase at each order of perturbation expansion. The value of the cut-off Λ is dictated by the consistency between one-loop calculations and the contribution of the higher dimensional operators. Typically, the theory becomes strongly interacting above the cut-off scale.

A physically important non-renormalizable effective theory is the theory of pions [2, 1]. The pions are pseudo-Goldstone bosons of the (approximate) global chiral symmetry $SU(2)_L \times SU(2)_R$ of strong interactions which is spontaneously broken down to $SU(2)_V$ of isospin. The physics of the light degrees of freedom (pions) is described by a non-linear σ-model. The chiral symmetry $SU(2)_L \times SU(2)_R$, non-linearly realized on the pion fields, requires non-renormalizable interaction. The lowest dimension one is

$$\mathcal{L}_{\text{pions}} = f_\pi^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) \tag{27}$$

where the fields

$$U = e^{i\pi^a \tau^a / f_\pi} \tag{28}$$

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transform under $SU(2)_L \times SU(2)_R$ linearly: $U \to V_L U V_R^\dagger$. The constanf f_π is the pion decay constant. The chiral symmetry cannot be reconciled with an effective renormalizable theory of pions and the Appelquist-Carazzone decoupling does not work.

The important question is: is the true low energy approximation to the more fundamental theory which explains the mechanism of electroweak symmetry breaking the renormalizable SM (like in supersymmetric extensions of the SM) or non-renormalizable electroweak theory (like in higgsless models with dynamical electroweak symmetry breaking and in models with the Higgs boson as a pseudo-Goldstone boson of some spontaneously broken bigger global symmetry)?

The predictive power and the phenomenological success of the SM suggests the first case. On the other hand, one may argue that the second option would more resemble spontaneous symmetry breaking in the condensed matter physics and in strong interactions.

3.2 Matter content and deeper unification?

There are two striking aspects of the matter spectrum in the Standard Model. One is the chiral anomaly cancellation [2, 1], which is necessary for a unitary (and renormalizable) theory, and occurs thanks to certain conspiracy between quarks and leptons suggesting a deeper link between them. The potential source of chiral anomalies in the Standard Model are the triangle diagrams like the ones shown in Figs. 4 in which now the external lines correspond to all possible triplets of currents coupled to the three types of gauge fields in the electroweak theory: $U(1)_Y$ gauge field B_μ, $SU(2)_L$ gauge fields W_μ^a ($a = 1, 2, 3$) and/or $SU(3)_C$ gauge fields A_μ^a ($a = 1, \ldots, 8$) and internal fermion lines correspond to all chiral fermions in the theory. Most of the anomaly coefficients vanish due to the group structure. The most interesting ones are the anomalies with one $U(1)_Y$ current and two $SU(2)_L$ currents and the one with three $U(1)_Y$ currents. They vanish (a necessary condition for consistency of the electroweak theory) provided

$$\sum Q_i = 0 \; , \tag{29}$$

(where Q_i's are the electric charges of the fields) separately for doublets and singlets of $SU(2)_L$. Incidently, the same condition is sufficient for vanishing of the mixed $U(1)_Y$-gravitational anomaly given by the diagrams like those shown in Figs. 4 but now with two currents corresponding to the energy-momentum tensors and the third one to the $U(1)_Y$ current. The condition (29) is satisfied in the SM because quark and lepton contributions cancel each other.

The second striking feature of the matter spectrum in the Standard Model is that it fits into simple representations of the $SU(5)$ and $SO(10)$ groups [3]. Indeed, we have, for $SU(5)$

$$\mathbf{5}^* = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \; d^c_L$$

$$\mathbf{10} = \begin{pmatrix} u \\ d_L \end{pmatrix}, \; u^c_L, \; e^c_L$$

$$\mathbf{1} = \nu^c_L \quad \text{(if the right} - \text{handed neutrino is added to the spectrum)}$$

and, for $SO(10)$,

$$\mathbf{16} = \mathbf{5}^* + \mathbf{10} + \mathbf{1}$$

The assignment of fermions to the $SU(5)$ representations fixes the normalization of the $U(1)_Y$ generator:

$$Q = T_3 + Y = L^{11} + \sqrt{\frac{5}{3}} L^{12} \; , \tag{30}$$

where L^{ij} are the $SU(5)$ generators satisfying the normalization condition $[L^{ij}, \; L^{kl}] = \frac{1}{2}\delta^{ik}\delta^{jl}$.

Both facts, the anomaly cancellation and the pattern of fermion spectrum, strongly suggest some kind of quark and lepton unification, at least at some very deep level, with some big group and some mechanism of its breaking. In addition, in line with the above conclusion is a well known fact that, with normalization given by eq (30), the running gauge couplings of the Standard Model approach each other at high scale of order 10^{13} GeV. Although unification of the gauge couplings in the Standard Model is only very approximate, it is nevertheless a remarkable fact that the strength of strong and electroweak interactions become comparable at certain energy scale.

3.3 Neutrino masses: evidence for new very high mass scale?

There is at present strong experimental evidence for neutrino oscillations whose most obvious and most natural explanation is that neutrinos are massive and their mass eigenstates are different from the weak interaction eigenstates. This is the first experimental evidence for physics beyond the Standard Model.

The smallness of the neutrino masses can be easily understood as due to the presence of a new very high mass scale M. If it makes sense to rely on

the Appelquist-Carazzone decoupling theorem then the mass scale M would manifest itself via higher dimension operators.

Neutrino mass terms may appear as dimension five operator

$$\frac{1}{M}(Hl_A)\lambda_{AB}(Hl_B) \tag{31}$$

where we use the following notation: $(Hl) \equiv \epsilon_{ij}H^i l^j$ denotes $SU(2)_L$ contraction and $ll \equiv \epsilon^{\alpha\beta}l_\alpha l_\beta$ denote Lorentz contraction. After spontaneous SM gauge symmetry breaking by the Higgs boson VEV the operator (31) gives indeed a Majorana mass matrix for neutrinos:

$$\mathcal{L}_{\nu\ \text{mass}} = -m_{AB}\ \nu_A\nu_B + \text{H.c.}\ , \qquad m_{AB} = \frac{v^2}{M}\lambda_{AB} \tag{32}$$

Small neutrino masses are obtained for big value of M, with the constants $\lambda_{AB} \sim \mathcal{O}(1)$. This is called a see-saw mechanism.

A possible and, in fact quite elegant, origin of the mass scale M would be the existence of another left-handed particle ν^c, a singlet of $SU(2)_L \times U(1)_Y$, i.e. a field such that

$$CP\nu^c(CP)^{-1} \equiv \nu_R \tag{33}$$

with a Majorana mass term

$$\mathcal{L}_{\text{Majorana}} = M_{\text{Maj}}^{AB}\ \nu_A^c \nu_B^c B + \text{H.c.} \tag{34}$$

It can be interpreted as a right-handed neutrino field. Moreover, we can construct Yukawa interactions

$$\epsilon_{ij}H_i\nu_B^c Y_\nu^{BA} l_j^A + \text{H.c.} \tag{35}$$

with a new set of (neutrino) Yukawa couplings Y_ν^{BA}. Both terms are $SU(2)_L \times U(1)_Y$ invariant and even renormalizable. We can consider then the diagram shown in Fig. 8. At the electroweak scale v, if $M_{Maj} \gg v$, we obtain the effective interaction shown in Fig. 9 described by the operator

$$\epsilon_{ij}H_i\ l_j^A\ Y_\nu^{DA}(M_{Maj}^{-1})^{DC}Y^{CB}\ \epsilon_{ij}H_i l_j^B \tag{36}$$

We recognize the previously introduced operator (31) with

$$\frac{\lambda_{AB}}{M} = (Y_\nu^T M_{MAJ}^{-1} Y_\nu)_{AB} \tag{37}$$

The see-saw mechanism with a new mass scale M is the most compelling explanation of the smallness of the neutrino masses [4, 5]. Indeed
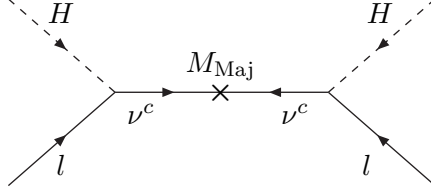
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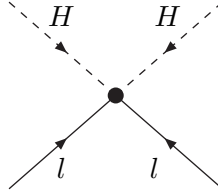Figure 8: Diagram generating the dimension 5 operator.

Figure 9: Effective dimension 5 operator.

i) the smallness of m_ν is then related to its zero electric charge

ii) the smallness of m_ν is also related to lepton number violation at the scale M

iii) with $m_\nu \sim Y^2 \frac{v^2}{M}$, $v = 240$ GeV and for $Y \sim \mathcal{O}(1)$ we get $m_\nu \sim (0.01 \div 0.1)$ eV for $M \sim (10^{15} \div 10^{13})$ GeV So, the scale M is close to the GUT scale.

iv) ν^c completes the spinor representation of S0(10)

v) heavy ν^c can play important role in baryogenesis via leptogenesis.

3.4 Hierarchy problem in the SM: hint for a new *low* mass scale?

Quantum corrections to the Higgs potential mass parameter m^2 in eq. (8) in the SM are quadratically divergent. If the SM is an effective low energy theory and has a cut-off at some mass scale M of new physics, $\Lambda_{SM} \sim M$, it means then that quantum corrections to m^2 are quadratically dependent on the new mass scale present in the underlying more fundamental theory. When $M \gg M_Z$ this is very unnatural even if m^2, that is M_Z, remains a free parameter of this underlying theory, and particularly difficult to accept

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if in the underlying theory m^2 is supposed to be fixed or indeed is fixed by some more fundamental considerations (as e.g. in supersymmetric and Little Higgs models, respectively). The latter is necessary if the underlying theory is to predict the scale of the electroweak symmetry breaking in terms of "more fundamental" parameters and, generically, in terms of its own cut-off Λ_{New}. Thus, for naturalness of the Higgs mechanism in the SM there should exist a new mass scale $M \gtrsim M_Z$, say only order of magnitude higher than M_Z and better understanding the mechanism of the electroweak symmetry breaking is, hopefully, a bridge to new physics that will be explored at the LHC.

For a more quantitative discussion of this so-called hierarchy problem we recall that in general in a field theory with a cut-off Λ and some scalar field(s) ϕ that can acquire VEV(s) the 1-loop effective potential is

$$\Delta V_{1-\text{loop}}(\phi) = \frac{1}{2} \int^\Lambda \frac{d^4 k}{(2\pi)^4} \, \text{STr} \ln \left[k^2 - \mathcal{M}^2(\phi) \right] = c\Lambda^4 + c'\Lambda^4 \ln \Lambda^2$$

$$+ \frac{1}{32\pi^2} \Lambda^2 \text{STr} \mathcal{M}^2(\phi) + \frac{1}{64\pi^2} \text{STr} \mathcal{M}^4(\phi) \ln \frac{\mathcal{M}^2(\phi)}{\Lambda^2} + \dots \quad (38)$$

where $\text{Str} \mathcal{M}^2(\phi) = \text{Tr}[(-)^F M^2(\phi)]$ with F - the fermion number operator and $\mathcal{M}^2(\phi)$ is the full ϕ-dependent mass matrix for all fields of the theory. The first terms in the expansion are the ϕ-independent contribution to the vacuum energy. We are interested in quantum corrections δm^2 to the mass parameter m^2 of the ϕ field potential. They are obtained by expanding

$$\text{STr} \mathcal{M}^2(\phi) = c_2 \phi^2 + \dots \,, \qquad \text{STr} \mathcal{M}^4(\phi) = c_4 \phi^2 + \dots \,, \qquad (39)$$

The corrections proportional to c_2 (to c_4) are in general quadratically (logarithmically) dependent on the cut-of scale Λ. In the SM with a cut-of Λ_{SM} we find

$$\delta m^2 = \left(\frac{\partial^2 \Delta V_{1-\text{loop}}(\phi)}{\partial \phi^2} \right)_{\text{min}} = \frac{3}{64\pi^2} \left(3g_2^2 + g_1^2 + \lambda - 8y_t^2 \right) \Lambda_{\text{SM}}^2 + \dots \,, (40)$$

If the SM was the correct theory up to the mass scale suggested by the see-saw mechanism, $\Lambda_{\text{SM}} \sim M_{GUT}$

$$|\delta m^2| \sim 10^{30} \text{ GeV}^2 \sim 10^{26} M_W^2 \,!$$

Clearly, this excludes the possibility of understanding the magnitude of Fermi constant $G_F \sim M_W^{-2}$ in any sensible way. We also see that for $|\delta m^2| \sim M_W^2, 10 M_W^2, 100 M_W^2$ one needs $\Lambda_{\text{SM}} \lesssim 0.5$ TeV, $\lesssim 1$ TeV, $\lesssim 6$ TeV,

respectively. If the scale of new physics is in the above range it should be discovered at the LHC.

However, for a solution of the hierarchy problem it is not enough to have a low physical cut-off scale of the SM. The deeper theory has its own cut-off scale Λ_{New} and the dependence on it of δm^2 calculated in this deeper theory should be mild enough, in order not to reintroduce the hierarchy problem for the electroweak scale.[2]

Many theoretical ideas have been proposed for solving the hierarchy problem of the electroweak scale. In supersymmetric extensions of the SM the dependence on their own cut-off scales Λ_{New} is only logarithmic because the quadratic divergences cancel out at any order of the perturbation expansion. Since the effective potential $V_{1-\text{loop}}(\phi)$ depends only on $\ln \Lambda_{\text{New}}$ (to any order of the perturbation expansion) the scale Λ_{New} can be as high as the Planck scale. The quadratic dependence on the SM cut-off scale Λ_{SM}, that is on the mass scale M_{SUSY} of the superparticles, shows up in

$$\text{STr}\mathcal{M}^4(\phi) = f(M_{\text{SUSY}}^2)\ \phi^2 + \dots \tag{41}$$

and more explicitly, at the 1-loop as[3]

$$\delta m^2 = \frac{1}{16\pi^2} \left(3g_2^2 + g_1^2 - 12y_t^2 \right) M_{\text{SUSY}}^2 \ln \frac{\Lambda_{\text{New}}^2}{M_{\text{SUSY}}^2}\ , \tag{42}$$

where we have replaced all soft supersymmetry breaking mass terms including the Higgs boson mass by M_{SUSY}.

Eq. (42) shows that in supersymmetric models the electroweak scale is calculable in terms of the known coupling constants and the (unknown) scales M_{SUSY} and Λ_{New}. For a natural solution to the hierarchy problem of the electroweak scale M_{SUSY} has to be low, say $M_{\text{SUSY}} \lesssim \mathcal{O}(10)M_W$. However, a new very difficult question appears about the hierarchy $\Lambda_{\text{New}}/M_{\text{SUSY}}$. This is the question about the mechanism of supersymmetry breaking. In gravity mediation scenarios $\Lambda_{\text{New}} \sim M_{\text{Pl}}$. In gauge mediation scenarios Λ_{New} is low but it is a new, introduced by hand, scale.

Other ideas for solving the hierarchy problem of the electroweak scale are more "pragmatic". Focusing on the scenarios with some predictive power, their general structure is the following: the low energy electroweak theory (but not necessarily the renormalizable SM) is embedded in a bigger one with a characteristic mass scale $\Lambda_{\text{SM}} \sim M \sim \mathcal{O}(1\ \text{TeV})$. The new physics is

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[2]The hierarchy of some other (new) scales is nevertheless usually present.

[3]The formula (42) applies in fact to $m_{H_2}^2$ which for $\tan \beta \gtrsim 5$ is the most important for electroweak symmetry breaking.

under perturbative control up to its cut-off $\Lambda_{\mathrm{New}} \gtrsim \mathcal{O}(10 \text{ TeV})$, high enough to avoid any conflict with precision electroweak data (to be discussed later). For such scenarios with $\Lambda_{\mathrm{New}} \gtrsim \mathcal{O}(10 \text{ TeV})$ to be useful for solving the electroweak hierarchy problem the dependence of δm^2 on Λ_{New} calculated in the extended theory has to be weak enough. This is obtained by ensuring that at least 1-loop contribution to the effective potential (38) have no quadratic dependence on Λ_{New}:

$$(\delta m^2)_{1-\mathrm{loop}} = 0 \cdot \Lambda_{\mathrm{New}}^2 + \mathcal{O}(\ln \Lambda_{\mathrm{New}}) + \mathrm{const.} \tag{43}$$

E.g. in the Little Higgs models the vanishing of the c_2 in $\mathrm{STr}\mathcal{M}^2(\phi)$ in eq. (39) is ensured by cancellation between contributions from particles of the same statistics. Such models predict the existence of new quark-like fermions and gauge bosons with masses $\sim M$. In these models, the quadratic dependence of δm^2 on Λ_{New} is present in higher order of the perturbation expansion but it is suppressed by loop factors. The tree level Higgs mass parameter m^2 usually vanishes $m^2(M)_{\mathrm{tree}} = 0$ as e.g. the Higgs boson is a Goldstone boson of some bigger (approximate) symmetry spontaneously broken at the scale M, i.e. M is identified with the "decay constant" and $\Lambda_{\mathrm{New}} \sim 4\pi M$. The electroweak symmetry is broken by quantum corrections. The electroweak scale is then predicted e.g. $M_W = M_W(\mathrm{couplings}, M, \Lambda_{\mathrm{New}})$ with a mild dependence on $\Lambda_{\mathrm{New}} \gtrsim \mathcal{O}(10 \text{ TeV})$ at which new unknown strong interactions set on. The crucial role is played by the new physics parameter M. In judging the plausibility of such ideas it is worth remembering our remarks in 3.1.

3.5 New low mass scale and precision electroweak data

The presence of new physics at low energy scale, $M \sim 1 \text{ TeV}$ raises the question on its contribution to the electroweak observables. We can address this question in a model independent way if again we assume the Appelquist-Carazzone decoupling scenario, i.e. renormalizable SM and corrections to it from new physics as higher dimension operators:

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \sum_{\hat{O}_i^{n+4}} \frac{c_i}{M^n} \hat{O}_i^{n+4} \ . \tag{44}$$

This time we are interested in operators which contribute to the electroweak observables. Such operators are necessarily of dimension $n \geq 6$. One can classify various contributions from new physics according to the value of the coefficients c_i in the Lagrangian (44): $c_i \sim \mathcal{O}(1)$ for new tree-level contributions or contributions from new strong interacting sector; $c_i \sim \mathcal{O}(1/16\pi^2)$

for contributions from perturbative new physics at 1-loop. Fitting the electroweak observables one obtains limits on $\frac{c_i}{M^2}$. Strictly speaking, the limits are applicable to new physics which gives renormalizable SM as its low energy limit but the results are also indicative for new mass scales in models like e.g. Little Higgs, in which the Appelquist-Carazzone decoupling does not work. In any case, the constraints on such models from the electroweak observables can be discussed model by model.

The task of using electroweak data to put limits on the new scale M is greatly facilitated by the expectation that the dominant corrections from new physics will show up as corrections to the gauge boson self-energies (the so-called oblique corrections). This expectation is based on the experience gained in the SM and with its various hypothetical extensions. Several authors have obtained limits on the scale M under plausible assumption that the main corrections to the SM fits appear in gauge boson self-energies

A comment is in order here. Gauge boson self-energies are not gauge invariant objects. The vertex and box contributions contain also pieces which are independent of the external legs. They cancel the gauge dependence of the full gauge boson self-energies and restore Ward-identity. Therefore, strictly speaking, there is an ambiguity in extracting the gauge boson self energies from fits to experimental data. Fortunately, the vertex and box contributions are in the SM in the commonly used gauges much smaller than the gauge boson self energies and the neglection of gauge dependence of the latters in obtaining experimental information about their magnitude seems to be a reasonable approach.

In the SM, with unbroken electromagnetic $U(1)$ gauge symmetry there are four independent gauge boson self-energies $\Pi_{ij}(q^2)$:

$$\int d^4x \ e^{-iq\cdot x}\langle J_i^\mu(x)J_j^\nu(0)\rangle = ig^{\mu\nu}\Pi_{ij}(q^2) + q^\mu q^\nu \text{ term} \qquad (45)$$

For instance, we can take $\Pi_{\gamma\gamma}$, $\Pi_{3\gamma}$, Π_{33} and Π_{11} as independent quantities ($i = 1,3$ are $SU(2)_L$ indices; the QED Ward identity implies $\Pi_{11}(q^2) = \Pi_{22}(q^2)$. In the limit of $q^2 \ll M^2$ we can expand

$$\begin{aligned}
\Pi_{\gamma\gamma}(q^2) &\approx q^2\Pi'_{\gamma\gamma}(0) \\
\Pi_{3\gamma}(q^2) &\approx q^2\Pi'_{3\gamma}(0) \\
\Pi_{33}(q^2) &\approx \Pi_{33}(0) + q^2\Pi'_{33}(0) \\
\Pi_{11}(q^2) &\approx \Pi_{11}(0) + q^2\Pi'_{11}(0)
\end{aligned} \qquad (46)$$

($\Pi_{\gamma\gamma}(0) = 0$ by QED Ward identity; the only non-zero contribution to $\Pi_{3\gamma}(0)$ comes from the $W^\pm$-charged Goldston boson loop). Thus, oblique

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corrections to the electroweak observables are to a good approximation parametrized by six constants. Three of them (or three linear combinations) are fixed in terms of α, M_Z and G_F by the renormalization procedure. In the remaining three combinations the UV divergences must cancel.[4] One usually defines [6, 7]

$$\alpha T \equiv \frac{e^2}{s^2 c^2 M_Z^2} \left[\Pi_{11}(0) - \Pi_{33}(0)\right] = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$
$$\alpha S \equiv 4e^2 \left[\Pi_{33}'(0) - \Pi_{3\gamma}'(0)\right] \propto \Pi_{3Y}'(0) \tag{47}$$
$$\alpha U \equiv 4e^2 \left[\Pi_{11}'(0) - \Pi_{33}'(0)\right]$$

It is clear from their definition that the parameters S, T and U have important symmetry properties: T and U vanish in the limit of unbroken custodial $SU(2)_V$ symmetry. The parameter S vanishes when $SU(2)_L$ is unbroken; unbroken $SU(2)_V$ is not sufficient for vanishing of S because $S \propto \Pi_{3Y}'(0) = \Pi_{3L,3R}'(0) + \Pi_{3L,B-L}'$ (the decomposition is labelled by the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum numbers) and $\mathbf{3}_L \times \mathbf{3}_R = \mathbf{1} + \mathbf{5}$ under $SU(2)_V$.

It turns out that in the SM the quantum corrections to the ρ parameter defined in section 2 as a ratio of physical (and measured) observables are to a very good approximation given by αT:

$$\Delta \rho \equiv \rho - 1 = \frac{M_W}{M_Z \cos\theta} - 1 \approx \alpha T \tag{48}$$

and, according to the eq. (18), depend quadratically on the top quark mass and logarithmically on the SM Higgs boson mass. Eq. (48) is a good approximation because other corrections to ρ (vertex corrections) are in the SM negligibly small. Similarly, in the SM

$$\alpha S = \frac{e^2}{48\pi^2} \left(-2\ln\frac{m_t^2}{M_Z^2} + \ln\frac{M_h^2}{M_Z^2}\right) . \tag{49}$$

It was discussed in section 2.3 that the SM quantum corrections agree excellently with electroweak data. The only free parameter in the fits of the SM to these data is the Higgs boson mass. The main part of this dependence enters through the ρ parameter and the data favour negligible contribution to ρ from $\ln(M_h/M_W)$ (see eq. (18)). The fits give $M_h \approx \mathcal{O}(100 \text{ GeV})$, with

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[4]The finitness of the gauge sector contribution to S, T and U requires the inclusion of the terms with $\Pi_{3\gamma}(0)$.

a big error since the dependence of the fits on the Higgs boson mass is only logarithmic. Such fits determine the values of parameters S_{SM}, T_{SM} and U_{SM} for the best fitted value of M_h.

We can discuss the room for new physics contribution to the electroweak fits by writing more generally:

$$
\begin{aligned}
T &= T_{SM}(M_h) + \Delta T \ , \\
S &= S_{SM}(M_h) + \Delta S \ , \\
U &= U_{SM}(M_h) + \Delta U \ ,
\end{aligned}
\tag{50}
$$

where $T_{SM}(M_h)$ etc. is the SM contribution for some fixed value of the Higgs boson mass.

A fit to the data gives now some values for ΔT, ΔS and ΔU as a function of the assumed value of M_h and shows how much room we have for new physics for different values of M_h. It is clear that for, say, $M_h = 115$ GeV such fits give ΔT, ΔS and ΔU consistent with zero and the only room for new physics is in the errors of the fitted values of these quantities. For larger values of M_h we have more room for new physics contributions. As we can see from eqs. (18) and (49) it must be positive to T and negative to S to balance the contribution of the larger Higgs boson mass. The fitted values of ΔT, ΔS and ΔU for different values of M_h give limits on the coefficients c_i/M of the dimension six operators that contribute to ΔT, ΔS and ΔU.

Several interesting conclusions have been reached in such studies [8]. First of all, independently of the assumed value of M_h, for $c_i \sim \mathcal{O}(1)$ the fits to the electroweak data give a lower limit $M \gtrsim \mathcal{O}(4 \text{ TeV})$ This limit is reached only for very correlated signs of the coefficients c_i. Thus, qualitatively speaking, any new physics with $M \lesssim \mathcal{O}(10 \text{ TeV})$ must be perturbative and cannot contribute at the tree level to be consistent with electroweak data. This strongly suggests a perturbative solution to the hierarchy problem.

Secondly, if new physics is indeed perturbative and shows up only at loop level, then the fits give $M_h \lesssim 240$ GeV independently of the value of the scale M.

Finally, if $M_h \gtrsim 300$ GeV then new physics with the scale $M \sim 10 \div 30$ TeV and with $c_i \sim \mathcal{O}(1)$ (i.e. strongly interacting) is actually needed! Thus, experimental discovery of the Higgs boson and the determination of its mass will be a strong hint about the kind of new physics one may expect.

4 Supersymmetric extensions of the Standard Model

In the rest of these lectures we discuss supersymmetric models as at present the most complete theoretical framework going beyond the Standard Model [9]. It has quite a few attractive features and also a number of difficulties, so that the full success of the SM is not automaticaaly recovered. We have already mentioned some of them in several places in these lectures but here we collect them together and extend our discussion. We shall not discuss one fundamental unresolved issue for supersymmetry which is the mechanism of spontaneous supersymmetry breaking and its transmission to the the SM sector. On the phenomenological side, the related problem is that of new and potentially dangerous sources in the soft supersymmetry breaking parameters of the FCNC and CP violating transitions. For a review of all these aspects of supersymmetric and supergravity models see e.g. [10, 11, 12].

4.1 Precision electroweak data

As discussed in section 3.4 supersymmetric models like the Minimal Supersymmetric Standard Model (MSSM) or its simple extensions satisfy a very important criterion of calculability. Most of the structure of the Standard Model is built into them, so the renormalizable Standard Model is their low energy approximation. Supersymmetric models are easily consistent [14] with the electroweak data since the supersymmetric quantum corrections to the Standard Model fits are suppressed by powers of the mass scale M_{SUSY} of supersymmetric particles and for $M_{SUSY} > \mathcal{O}(500)$ GeV are well below experimental errors (in particular, the custodial symmetry breaking by the sfermion masses is sufficiently suppressed). Thus, the predictive power of the Standard Model remains intact and its success is not accidental.

4.2 The electroweak symmetry breaking

Supersymmetric models solve the hierarchy problem of the electroweak scale. In the limit of unbroken supersymmetry the quadratically divergent quantum corrections to the Higgs mass parameter are absent in any order of perturbation theory. When supersymmetry is softly broken by a mass scale M, the superpartners get their masses from the electroweak breaking and from the supersymmetry breaking mass terms $\sim M$. They decouple at energies smaller than M and the quadratically divergent Standard Model contribution to the Higgs mass parameter is cut-off by M and, therefore, depends quadratically on M. Thus, the hierarchy problem of the electroweak scale disappears if $M \lesssim \mathcal{O}(1)$ TeV. The cut-off to a supersymmetric theory

can be as high as the Planck scale and "small" scale of the electroweak symmetry breaking is still natural.

The electroweak symmetry breaking may be triggered by radiative corrections to the Higgs potential:

$$(\delta m^2_{H_2})_{1-\text{loop}} \sim -\mathcal{O}(0.1) \, M^2 \ln \frac{\Lambda}{M} \tag{51}$$

This formula follows from eq. (42). If we assume that $m^2_0 \sim M^2$, i.e. that the tree level Higgs mass parameter is approximately equal to the soft supersymmetry breaking scale the radiative electroweak symmetry breaking $((m^2_{H_2})_{1-\text{loop}} < 0)$ is triggered by the large top quark Yukawa coupling, hidden in the numerical factors of eq. (51). With the Higgs boson self-interactions fixed by the gauge couplings of the Standard Model

$$\lambda \phi^4 \rightarrow g^2 \phi^4 \tag{52}$$

one obtains the correct prediction for the electroweak scale for $\Lambda \sim M_{GUT, \, Pl}$. This nicely fits with unification of the gauge couplings.

4.3 The mass of the lightest Higgs boson

Supersymmetric models typically restrict the couplings in the Higgs potential and give strong upper bounds on the mass of the lightest Higgs particle [13]. In the minimal model the Higgs boson self-coupling comes from the D-terms and its self-coupling is the gauge coupling, eq.(52). Therefore, at the tree level

$$M_{\text{Higgs}} < M_Z \approx 91 \text{ GeV} \tag{53}$$

There are large quantum corrections to this result. They depend quadratically on the top quark mass and logarithmically on the stop mass scale $M_{\tilde{t}} \sim M_{\text{SUSY}}$:

$$M^2_{\text{Higgs}} = \lambda v^2 \tag{54}$$

where λ is given by

$$\lambda = \frac{1}{8}(g^2_2 + g^2_1)\cos^2 2\beta + \Delta\lambda \, , \qquad \text{with} \quad \Delta\lambda = \frac{3g^2_2}{8\pi^2} \frac{m^4_t}{v^2 M^2_W} \ln \frac{M^2_{\tilde{t}}}{m^2_t} \, . \tag{55}$$

The present experimental limit $M_{\text{Higgs}} > 114$ GeV requires $M_{\tilde{t}} \gtrsim 500$ GeV and for $M_{\tilde{t}} < 1$ TeV, $M_{\text{Higgs}} < 130$ GeV. The closer the Higgs mass would be to the present experimental limit, the better it would be for the "naturalness" of the electroweak scale. Clearly, in the MSSM, the tunning in

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the Higgs potential depends exponentially on the Higgs mass and one may eventually have some tension here (see eqs. (55), (42)).

One can depart from the minimal model and relax the bound on the Higgs mass. For instance, with an additional chiral superfield which is a Standard Model singlet, one may couple the singlet to the Higgs doublets and get additional contributions to the Higgs self-coupling. Explicit calculations show that in such and other models, with $M \lesssim 1$ TeV, the bound on the Higgs mass cannot be raised above ~ 150 GeV if one wants to preserve perturbative gauge coupling unification.

4.4 Gauge coupling unification

It is well known [15] that in the framework of the MSSM with degenerate sparticle spectrum characterized by $M_{SUSY} \approx 1$ TeV the three experimentally measured gauge couplings unify with high precision at the scale $M_{GUT} \sim 10^{16}$ GeV. This gives support to perturbative new physics at $\mathcal{O}(1 \text{ TeV})$. Supersymmetry and the idea of grand unification (see section 3.2) mutually strengthen their attractiveness.

A closer look at the unification is interesting. One may ask how precise is the unification when the superpartner masses are not degenerate and different from 1 TeV. It has been understood that even for nondegenerate superpartner spectrum the superpartner mass dependence of the RG evolution of the gauge couplings can be described to a very good approximation by a single effective parameter T. The superparticle threshold effects are correctly included in the supersymmetric 1-loop RGE whose running starts at T, with the SM RG equations used below the scale T. For consistency, 2-loop running should also be included.

T depends strongly on the higgsino (μ) and gaugino (M_i) mass parameters and much weaker on the sfermion masses. Exact unification of the measured gauge couplings requires $T \approx 1$ TeV, i.e. the higgsino and the gaugino physical masses ~ 1 TeV if degenerate. However, a more plausible assumption that the parameters μ and M_i are approximately degenerate and ~ 1 TeV at the GUT scale gives $T \approx 100$ GeV because of strong renormalization effects. Thus, a realistic spectrum does not give exact unification and one may wonder about the accuracy of unification in the MSSM.

In order to define what we understand by 'successful unification' let us first recall the one-loop renormalization group equations in the SM and MSSM. At one-loop the gauge couplings $\tilde{\alpha}_i$ of the three group factors of

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G_{SM} run according to the equations:

$$\frac{1}{\tilde{\alpha}_i(Q)} = \frac{1}{\tilde{\alpha}_i(M_Z)} - \frac{b_0^{(i)}}{2\pi} \ln\left(\frac{Q}{M_Z}\right) + \delta_i \tag{56}$$

Here, $1/\tilde{\alpha}_i(M_Z) = (58.98 \pm 0.04,\ 29.57 \pm 0.03,\ 8.40 \pm 0.14)$ are the experimental values of the gauge couplings at the Z^0-pole and $b_0^{(i)}$ are the one-loop coefficients of the relevant beta-functions. They read $b_0 = (\frac{1}{10} + \frac{4}{3}N_g, -\frac{43}{6} + \frac{4}{3}N_g, -11 + \frac{4}{3}N_g)$ in the SM and $b_0 = (\frac{3}{5} + 2N_g, -5 + 2N_g, -9 + 2N_g)$ in the MSSM, where N_g is the number of generations. Threshold corrections (e.g. from heavy GUT gauge bosons) are represented by the parameters δ_i. As explained earlier using the MSSM RG equations directly from the electroweak scale for $T \approx M_Z$ means that the supersymmetric threshold corrections corresponding to a realistic mass spectrum are properly included.

In the bottom-up approach one can speak about the gauge coupling unification if in some range of scales Q the couplings defined by eq. (56) with, in general, Q-dependent $\delta_i(Q)$ can take a common value $\alpha_i(Q) = \alpha_{\mathrm{GUT}}$ for reasonably small values of $\delta_i(Q)$ (compared to $\alpha_{\mathrm{GUT}}^{-1}$).[5] The condition for the unification can be succintly written as

$$\epsilon_{ijk}\left(\frac{1}{\tilde{\alpha}_i(M_Z)} + \delta_i\right)(b_0^{(j)} - b_0^{(k)}) = 0 \tag{57}$$

Putting in the experimental values for $\alpha_i(M_Z)$ and the beta-function coefficients we get:

$$-41.1 + 3.8\delta_1 - 11.1\delta_2 + 7.3\delta_3 = 0 \qquad \text{(SM)}$$
$$-0.9 + 4\delta_1 - 9.6\delta_2 + 5.6\delta_3 = 0 \qquad \text{(MSSM)} \tag{58}$$

We see, that to achieve the gauge coupling unification at the one-loop level we need the threshold corrections δ_i to be of order 10% $\alpha_{\mathrm{GUT}}^{-1}$ in the SM, while in the MSSM we need only $\delta_i \sim 1\%$ $\alpha_{\mathrm{GUT}}^{-1}$. In the MSSM once the two loop effects are inluded one needs δ_i's by factor 2 larger. The unification of the gauge cuplings in the MSSM is indeed very precise: it admits (and requires) only 2% threshold corrections from the GUT physics. These 2% corrections give 10% effect on α_s at M_Z scale, but the precision of unification in the MSSM should be judged by the necessary for exact unification threshold corrections at the GUT scale.

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[5]Whether there exists a unified model able to provide such values of $\delta_i(Q)$'s is a different question.

Unification of the gauge couplings does not necessarily imply the standard GUT theories with all their problems, like spontaneous breaking of the GUT gauge group by VEVs of some Higgs fields, the doublet-triplet splitting problem, etc. Many different solutions have been proposed.

With the threshold corrections of the right order of magnitude, the unification scale can be estimated from the equation:

$$\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} - \frac{1}{2\pi}(b_0^{(1)} - b_0^{(2)}) \ln\left(\frac{M_{\mathrm{GUT}}}{M_Z}\right) + (\delta_1 - \delta_2) = 0 \quad (59)$$

For the sake of concreteness, we assume here that $\delta_1 = \delta_2 = 0$ and that all threshold corrections are accounted for by δ_3 (thus, the unification point is assumed to be where α_1 and α_2 intersect). Putting in the experimental numbers and the beta-function coefficients we get $M_{\mathrm{GUT}} \approx 1 \times 10^{13}$ GeV in the SM and $M_{\mathrm{GUT}} \approx 2 \times 10^{16}$ GeV in the MSSM.

The scale of unification in the MSSM is determined very precisely to be in the range $(2 \div 4) \times 10^{16}$ GeV. This is interesting because it is very close to the reduced Planck scale $M_{\mathrm{Pl}} = 2 \times 10^{18}$ GeV and could be considered as evidence for unification including gravity.[6] But one to two orders of magnitude difference between the two scales needs some explanation. Of course, new particles in incomplete $SU(5)$ representations would alter the running and could push the unification scale closer to the Planck scale. However, one must not destroy the precision of the unification by new threshold corrections, so this possibility looks very fine-tuned and *ad hoc*. An interesting possibility would be to unify the three gauge interactions with gravity at $M_{GUT} \sim 10^{16}$ GeV by changing the energy dependence of the gravity coupling. This is possible if gravitational interactions (and only they) live in more than 3 spatial dimentions. The effective four-dimensional Planck constant is then

$$M_{\mathrm{Pl}}^2 = M_{\mathrm{Pl}\ (4+n)}^{2+n} R^n \ , \qquad (60)$$

where n is the number of extra dimensions, R is their compactification radius and $M_{\mathrm{Pl}\ (4+n)}^{2+n}$ is the Planck scale in $4+n$ dimensions which we would like to take equal to $M_{GUT} \sim 10^{16}$ GeV. For $n = 1$ (like in the M-theory of Horava and Witten) we get $1/R \sim 10^{14}$ GeV.

4.5 Proton decay

[6]In string theories without the stage of Grand Unification below the compactification scale M_S the couplings unify at M_S which e.g. in weakly coupled heterotic string is about factor 5 below M_{Pl}.

In the SM the baryon number is (perturbatively) conserved since there are no renormalizable couplings violating this symmetry (see section. 2.5). Experimental search for proton decay, e.g $p \to e^+\pi^0$, $p \to K^+\nu$ is one of the most fundamental tasks for particle physics. The present limit is $\tau_p > 10^{33}$ years. If the SM is only an effective low energy theory the remnants of new physics should show up as non-renormalizable corrections to the SM. The lowest dimension operators for the proton decay (with the particle spectrum of the SM) is the set of dimension 6 operators of the form

$$\hat{O}_i^{(6)} = \frac{c_i^{(6)}}{M_{(6)}^2} qqql \tag{61}$$

For such operators for the proton lifetime we get

$$\frac{1}{\tau_p} = \frac{[c_i^{(6)}]^2}{16\pi} \frac{m_p^5}{M_{(6)}^4}$$

The limit $\tau_p > 10^{33}$ years gives then $M_{(6)} > \sqrt{c_i^{(6)}} \times 10^{16}$ GeV (this is only a very rough estimate which neglects strong interaction effects). Any new physics with lower mass scale that could lead to proton decay should be coupled with $c_i^{(6)} \ll 1$. For instance, for $c_i^{(6)} \sim \alpha_{\mathrm{GUT}} \approx 1/25$ we get $M_{(6)} \gtrsim \times 10^{15}$ GeV which is still too high for the SM unification ($M_{\mathrm{GUT}} \approx 10^{13}$ GeV).

In supersymmetric extensions of the SM, with softly broken low energy supersymmetry there are low mass scalars in the spectrum with masses $M \sim \mathcal{O}(1 \text{ TeV})$, which may have renormalizable couplings to quarks and leptons.

Indeed, the most general renormalizable superpotential in the minimal supersymmetric model is

$$\begin{aligned} w = &\hat{U}^c\hat{Q}\hat{H}_u + \hat{D}^c\hat{Q}\hat{H}_d + \hat{E}^c\hat{L}\hat{H}_d + \hat{H}_d\hat{H}_u \\ &+ \hat{D}^c\hat{Q}\hat{L} + \hat{E}^c\hat{L}\hat{L} + \hat{U}^c\hat{D}^c\hat{D}^c + \hat{L}\hat{H}_u \ , \end{aligned} \tag{62}$$

(the coupling constants and the flavour indices are suppressed). The second line is also consistent with the SM gauge symmetry but these interactions do not conserve baryon and lepton numbers and give renormalizble couplings of scalars to fermions. After integrating the scalars out one gets dimension 6 operators as in (61) with $M_{(6)} \sim M_{\mathrm{SUSY}}$ from diagrams shown in Fig. 10 and to be consistent with the limit on the proton lifetime we need $c^{(6)} = \lambda_1\lambda_2 < 10^{-26}$.

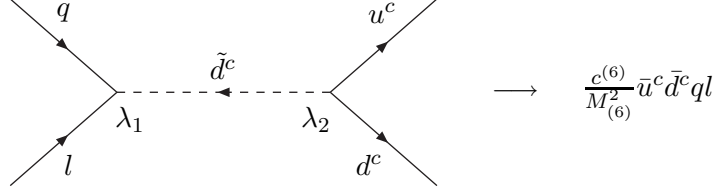$$\longrightarrow \quad \frac{c^{(6)}}{M_{(6)}^2} \bar{u}^c \bar{d}^c q l$$

Figure 10: Diagram generating the dimension 6 operator.

One can forbid the terms in the second line of eq. (62) by imposing a discrete symmetry, the so-called matter parity $R_p = (-1)^{3(B-L)}$. Such a symmetry could for instance, be a discrete remnant of the gauged $U(1)_{B-L}$ in the $SO(10)$ theory [17]. Matter parity is equivalent to R-parity $R = (-1)^{2S+3(B-L)}$ acting on the component fields, where S is their spin, since Lorentz-invariant interactions preserve $(-1)^{2S}$. We get then a stable LSP - candidate for dark matter in the Universe.

In supersymmetric GUT models, even with $R-$parity imposed, there is still another source of dangerous contributions to the proton decay amplitudes. These are the dimension 5 operators

$$\hat{O}_i^{(5)} = \frac{c_i^{(5)}}{M_{(5)}^2} q q \tilde{q} \tilde{l} \tag{63}$$

which when inserted into one loop diagrams with gaugino exchanges give rise via diagrams shown in Fig. 11 to dimension 6 operators. In the effec-
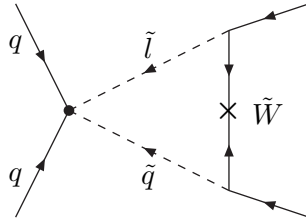

Figure 11: Loop diagram generating the dimension 6 operator form the dimension 5 operator.

tive dimension 6 operator of the form (61) one then gets $c^{(6)} = \alpha_{\mathrm{GUT}} c^{(5)}$,

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$M_{(6)}^2 = M_{(5)}M_{\text{SUSY}}$. From $\tau_p > 10^{33}$ years and for $M_{\text{SUSY}} \sim \mathcal{O}(1 \text{ TeV})$, $M_{(5)} \sim M_{\text{GUT}} \sim 10^{16}$ GeV, one gets $c^{(5)} \lesssim 10^{-7}$. Thus we need small couplings in the amplitudes generating the dimension 5 operators. In SUSY GUT's dimension 5 operators originate from the exchange of the colour triplet scalars present in the Higgs boson GUT multiplets as shown in Fig. 12, so $c^{(5)} \sim Y^2$ (Yukawa couplings for the quarks of the first two
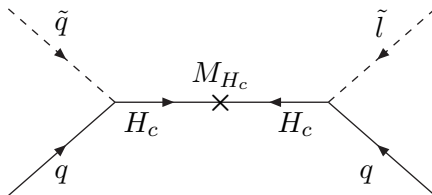
Figure 12: Diagram generating the dimension 5 operator for proton decay.

generations) and $M_{(5)} \sim M_{H_c} \sim M_{\text{GUT}}$.

Given various uncertainties, e.g. in the unknown squark, gaugino and heavy Higgs boson mass spectrum, such contributions in supersymmetric GUT models are marginally consistent with the experimental limits on the proton lifetime, particularly in models more complicated than the minimal supersymmetric $SU(5)$ model. Concrete classical GUT models are, however, not very attractive and plagued with various problems like e.g. doublet-triplet splitting problem for the Higgs boson multiplets. There are several interesting other ideas like unification in (small) extra dimensions or in string theory [18], which offer the possibility of avoiding those difficulties and simultaneously preserving the attractive features of GUT's. In some of such models proton is stable or its lifetime makes its decays unobservable experimentally. An interesting question is: what if proton after all decays but slow enough to rule out classical GUT models?

5   Summary

Thinking about new physics from the perspective of the extremely succesful Standard Model is very challenging. At present there is no approach that fully and convincingly incorporates this succes into its structure. Focusing on the electroweak symmetry breaking alone, one may wonder if the high predictive power of the renormalizable SM for the electroweak observables

and its perfect agreement with experimental data is significant or partly accidental. If significant - it supports supersymmetry; if partly accidental - we have more room for various speculations about the mechanism of electroweak symmetry breaking. Experiments are needed to put us on the right track, and hopefully, experiments at the LHC will give us the necessary insight.

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